Shear Reinforcement for Slabs

Reported by Joint ACI-ASCE Committee 421

Thomas C. Schaeffer
Chairman

Carl H. Moon
Secretary

Scott D. B. Alexander
Pinaki R. Chakrabarti
William L. Gamble
Amin Ghali*
Hershell Gill
Neil L. Hammill
J. Leroy Hulsey
Theodor Krauthammer*
James S. Lai
Mark D. Marvin
Edward G. Nawy
Eugenio M. Santiago
Sidney H. Simmonds
Miroslav F. Vejvoda
Stanley C. Woodson*

*Subcommittee members who were involved in preparing this report.

Tests have established that punching shear in slabs can be effectively resisted by reinforcement consisting of vertical rods mechanically anchored at top and bottom of slabs. ACI 318 sets out the principles of design for slab shear reinforcement and makes specific reference to stirrups and shear heads. This report reviews other available types and makes recommendations for their design. The application of these recommendations is illustrated through a numerical example.

Keywords: column-slab junction; concrete design; design; moment transfer; prestressed concrete; punching shear; shearheads; shear stresses; shear studs; slabs; two-way floors.

CONTENTS

Notation, p. 421.1R-2

Chapter 1—Introduction, p. 421.1R-2
1.1—Objectives
1.2—Scope
1.3—Evolution of the practice

Chapter 2—Role of shear reinforcement, p. 421.1R-3

Chapter 3—Design procedure, p. 421.1R-3
3.1—Strength requirement
3.2—Calculation of factored shear stress v_u
3.3—Calculation of shear strength v_n
3.4—Design procedure

Chapter 4—Prestressed slabs, p. 421.1R-6
4.1—Nominal shear strength

Chapter 5—Suggested higher allowable values for v_c, v_n, s, and f_{yv}, p. 421.1R-6
5.1—Justification
5.2—Value for v_c
5.3—Upper limit for v_n
5.4—Upper limit for s
5.5—Upper limit for f_{yv}

Chapter 6—Tolerances, p. 421.1R-6

Chapter 7—Design example, p. 421.1R-7

Chapter 8—Requirements for seismic-resistant slab-column in regions of seismic risk, p. 421.1R-8

Chapter 9—References, p. 421.1R-9
9.1—Recommended references
9.2—Cited references

ACI 421.1R-99 became effective July 6, 1999.

Appendix A—Details of shear studs, p. 421.1R-10
A.1—Geometry of stud shear reinforcement
A.2—Stud arrangements
A.3—Stud length

Appendix B—Properties of critical sections of general shape, p. 421.1R-11

Appendix C—Values of v_c within shear reinforced zone, p. 421.1R-13

NOTATION

A_c = area of concrete of assumed critical section
A_v = cross-sectional area of the shear studs on one peripheral line parallel to the perimeter of the column section
b_o = perimeter of critical section
c_b, c_t = clear concrete cover of reinforcement to bottom and top slab surfaces, respectively.
c_x, c_y = size of a rectangular column measured in two orthogonal span directions
d = effective depth of slab
d_b = nominal diameter of flexural reinforcing bars
D = stud diameter
f'_c = specified compressive strength of concrete
f_{ct} = average splitting tensile strength of lightweight aggregate concrete
f_{pc} = average value of compressive stress in concrete in the two directions (after allowance for all prestress losses) at centroid of cross section
f_{yv} = specified yield strength of shear studs
h = overall thickness of slab
I_x, I_y = second moment of area of assumed critical section about the axis x and y
J_x, J_y = property of assumed critical section analogous to polar moment of inertia about the axes x and y
l = length of a segment of the assumed critical section
l_x, l_y = projections of assumed critical section on principal axes x and y
l_{x1}, l_{y1} = length of sides in the x and y directions of the critical section at $d/2$ from column face
l_{x2}, l_{y2} = length of sides in the x and y directions of the critical section at $d/2$ outside the outermost studs
l_s = length of stud (including top anchor plate thickness; see Fig. 7.1)
M_{ux}, M_{uy} = factored unbalanced moments transferred between the slab and the column about centroidal axes x and y of the assumed critical section
n_x, n_y = numbers of studs per line/strip running in x and y directions
s = spacing between peripheral lines of studs
s_o = spacing between first peripheral line of studs and column face
v_c = nominal shear strength provided by concrete in presence of shear studs
v_n = nominal shear strength at a critical section
v_s = nominal shear strength provided by studs
v_u = maximum shear stress due to factored forces
V_p = vertical component of all effective prestress forces crossing the critical section
V_u = factored shear force
x, y = coordinates of the point at which v_u is maximum with respect to the centroidal principal axes x and y of the assumed critical section
α = distance between column face and a critical section divided by d
α_s = dimensionless coefficient equal to 40, 30, and 20 for interior, edge and corner columns, respectively
β_c = ratio of long side to short side of column cross section
β_p = constant used to compute v_c in prestressed slabs
γ_{vx}, γ_{vy} = fraction of moment between slab and column that is considered transferred by eccentricity of the shear about the axes x and y of the assumed critical section
ϕ = strength-reduction factor = 0.85

CHAPTER 1—INTRODUCTION

1.1—Objectives

In flat-plate floors, slab-column connections are subjected to high shear stresses produced by the transfer of axial loads and bending moments between slab and columns. Section 11.12.3 of ACI 318 allows the use of shear reinforcement for slabs and footings in the form of bars, as in the vertical legs of stirrups. ACI 318R emphasizes the importance of anchorage details and accurate placement of the shear reinforcement, especially in thin slabs. A general procedure for evaluation of the punching shear strength of slab-column connections is given in Section 11.12 of ACI 318.

Shear reinforcement consisting of vertical rods (studs) or the equivalent, mechanically anchored at each end, can be used. In this report, all types of mechanically-anchored shear reinforcement are referred to as "shear stud" or "stud." To be fully effective, the anchorage must be capable of developing the specified yield strength of the studs. The mechanical anchorage can be obtained by heads or strips connected to the studs by welding. The heads can also be formed by forging the stud ends.

1.2—Scope

These recommendations are for the design of shear reinforcement using shear studs in slabs. The design is in accordance with ACI 318, treating a stud as the equivalent of a vertical branch of a stirrup. A numerical design example is included.

1.3—Evolution of the practice

Extensive tests[1-6] have confirmed the effectiveness of mechanically-anchored shear reinforcement, such as shown in Fig. 1.1,[7] in increasing the strength and ductility of slab-column connections subjected to concentric punching or punching combined with moment. The Canadian Concrete Design Code (CSA A23.3) and the German Construction Supervising Authority, Berlin,[8] allow the use of shear studs for flat slabs (Fig. 1.2). Design rules have been presented[9] for application of British Standard BS 8110 to stud design for slabs. Various

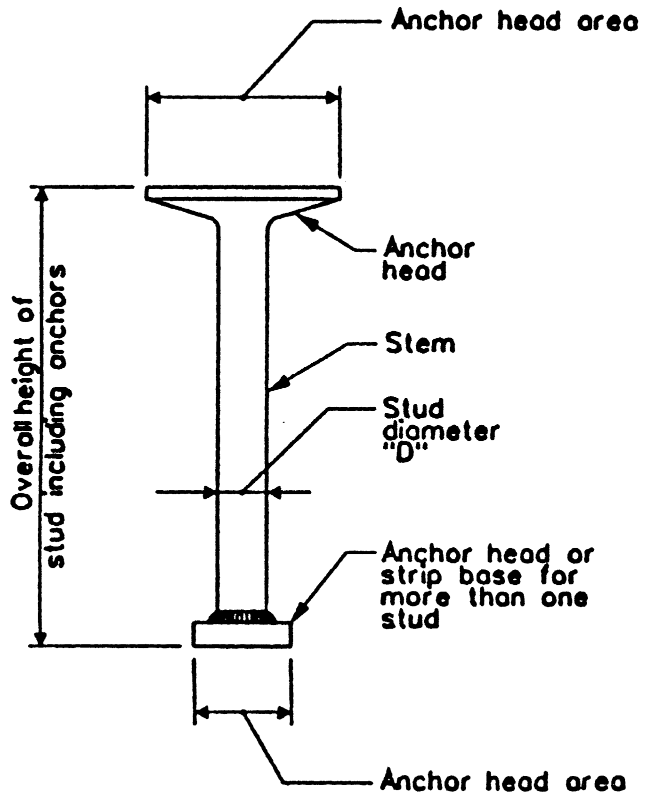


Fig. 1.1—Shear stud assembly.

forms of such devices were applied and tested by other investigators, as described in Appendix A.

CHAPTER 2—ROLE OF SHEAR REINFORCEMENT

Shear reinforcement is required to intercept shear cracks and prevent them from widening. The intersection of shear reinforcement and cracks can be anywhere over the height of the shear reinforcement. The strain in the shear reinforcement is highest at that intersection.

Effective anchorage is essential and its location must be as close as possible to the structural member's outer surfaces. This means that the vertical part of the shear reinforcement must be as tall as possible to avoid the possibility of cracks passing above or below it. When the shear reinforcements are not as tall as possible, they may not intercept all inclined shear cracks. Anchorage of shear reinforcement in slabs is achieved by mechanical ends (heads), bends, and hooks. Tests[1] have shown, however, that movement occurs at the bends of shear reinforcement, at Point A of Fig. 2.1, before the yield strength can be reached in the shear reinforcement, causing a loss of tension. Furthermore, the concrete within the bend in the stirrups is subjected to stresses that could exceed 0.4 times the stirrup's yield stress, f_{yv}, causing concrete crushing. When f_{yv} is 60 ksi (400 MPa), the average compressive stress on the concrete under the bend can reach 24 ksi (160 MPa) and local crushing can occur. These difficulties, including the consequences of improper stirrup details, have also been discussed by others.[10-13] The movement at the end of the vertical leg of a stirrup can be reduced by attachment to a flexural reinforcement bar as shown, at Point B of Fig. 2.1.

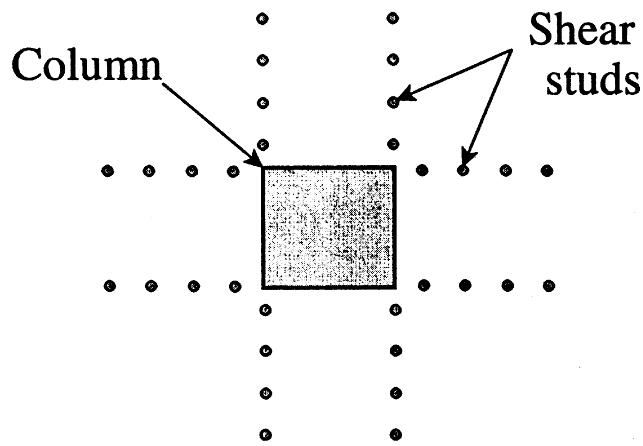


Fig. 1.2—Top view of flat slab showing locations of shear studs in vicinity of interior column.

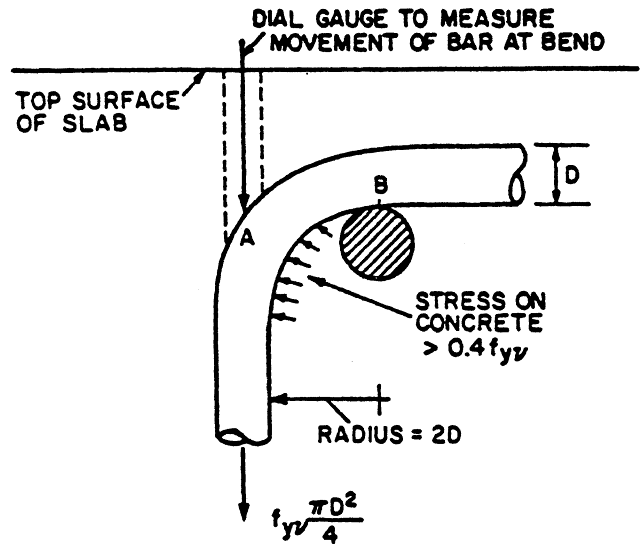


Fig. 2.1—Geometrical and stress conditions at bend of shear reinforcing bar.

The flexural reinforcing bar, however, cannot be placed any closer to the vertical leg of the stirrup, without reducing the effective slab depth, *d*. Flexural reinforcing bars can provide such improvement to shear reinforcement anchorage only if attachment and direct contact exists at the intersection of the bars, Point B of Fig. 2.1. Under normal construction, however, it is very difficult to ensure such conditions for all stirrups. Thus, such support is normally not fully effective and the end of the vertical leg of the stirrup can move. The amount of movement is the same for a short or long shear reinforcing bar. Therefore, the loss in tension is important and the stress is unlikely to reach yield in short shear reinforcement (in thin slabs). These problems are largely avoided if shear reinforcement is provided with mechanical anchorage.

CHAPTER 3—DESIGN PROCEDURE

3.1—Strength requirement

This chapter presents the design procedure for mechanically-anchored shear reinforcement required in the slab in the vicinity of a column transferring moment and shear. The requirements of ACI 318 are satisfied and a stud is treated as

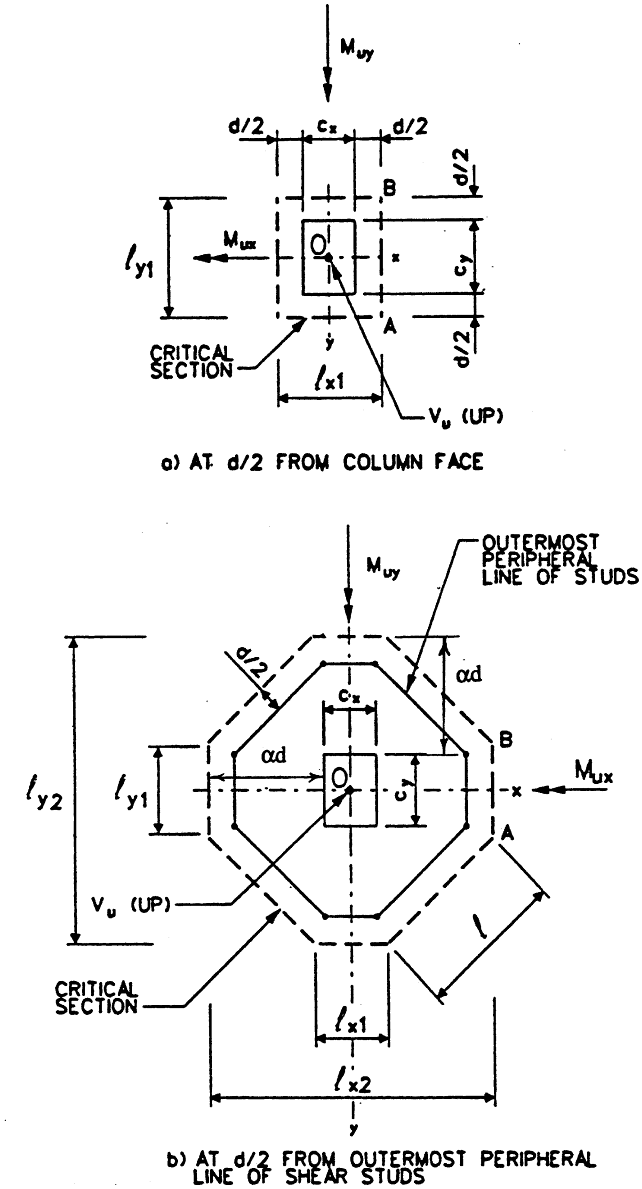


Fig. 3.1—Critical sections for shear in slab in vicinity of interior column.

the equivalent of one vertical leg of a stirrup. The equations of Section 3.3.2 apply when conventional stirrups are used. The shear studs shown in Fig. 1.2 can also represent individual legs of stirrups.

Design of critical slab sections perpendicular to the plane of a slab should be based upon

$$v_u \leq \phi v_n \tag{3.1}$$

in which v_u is the shear stress in the critical section caused by the transfer between the slab and the column of factored axial force or factored axial force combined with moment; and v_n is the nominal shear strength (Eq. 3.5 to 3.9).

Eq. 3.1 should be satisfied at a critical section perpendicular to the plane of the slab at a distance *d/2* from the column perimeter and located so that its perimeter b_o, is minimum but need not approach closer than *d/2* to the outermost peripheral line of shear studs.

3.2—Calculation of factored shear strength v_u

The maximum factored shear stress v_u at a critical section produced by the combination of factored shear force V_u and unbalanced moments M_{ux} and M_{uy}, is given in Section R11.12.6.2 of ACI 318R:

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_{vx} M_{ux} y}{J_x} + \frac{\gamma_{vy} M_{uy} x}{J_y} \tag{3.2}$$

in which

A_c = area of concrete of assumed critical section

x, y = coordinate of the point at which v_u is maximum with respect to the centroidal principal axes x and y of the assumed critical section

M_{ux}, M_{uy} = factored unbalanced moments transferred between the slab and the column about the centroidal axes x and y of the assumed critical section, respectively

γ_{ux}, γ_{uy} = fraction of moment between slab and column that is considered transferred by eccentricity of shear about the axes x and y of the assumed critical section. The coefficients γ_{ux} and γ_{uy} are given by:

$$\gamma_{vx} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{l_{y1}/l_{x1}}}; \tag{3.3}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{l_{x1}/l_{y1}}}$$

where l_{x1} and l_{y1} are lengths of the sides in the x and y directions of the critical section at *d/2* from column face (Fig. 3.1a).

J_x, J_y = property of assumed critical section, analogous to polar amount of inertia about the axes x and y, respectively.

In the vicinity of an interior column, J_y for a critical section at *d/2* from column face (Fig. 3.1a) is given by:

$$J_y = d\left[\frac{l_{x1}^3}{6} + \frac{l_{y1} l_{x1}^2}{2}\right] + \frac{l_{x1} d^3}{6} \tag{3.4}$$

To determine J_x, interchange the subscripts x and y in Eq. (3.4). For other conditions, any rational method may be used (Appendix B).

3.3—Calculation of shear strength v_n

Whenever the specified compressive strength of concrete f_c' is used in Eq. (3.5) to (3.10), its value must be in lb per in.^{2}. For prestressed slabs, see Chapter 4.

3.3.1 *Shear strength without shear reinforcement*—For nonprestressed slabs, the shear strength of concrete at a critical

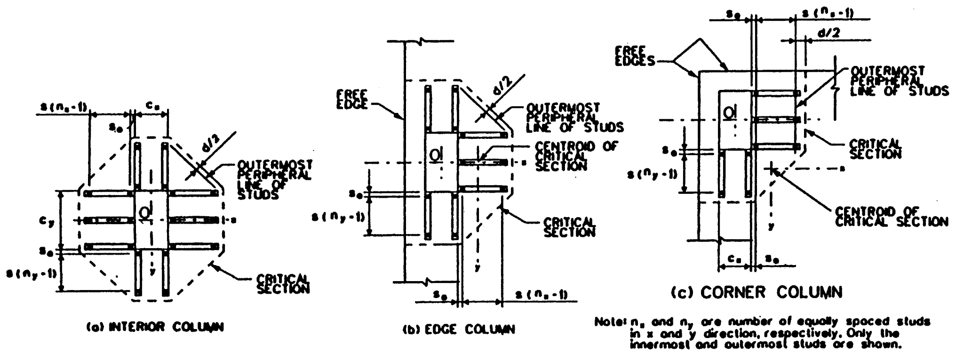


Fig. 3.2—Typical arrangement of shear studs and critical sections outside shear-reinforced zone.

section at $d/2$ from column face where shear reinforcement is not provided should be the smallest of:

$$\text{a) } v_n = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f'_c} \qquad (3.5)$$

where β_c is the ratio of long side to short side of the column cross section.

$$\text{b) } v_n = \left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f'_c} \qquad (3.6)$$

where α_s is 40 for interior columns, 30 for edge columns, 20 for corner columns, and

$$\text{c) } v_n = 4\sqrt{f'_c} \qquad (3.7)$$

At a critical section outside the shear-reinforced zone,

$$v_n = 2\sqrt{f'_c} \qquad (3.8)$$

Eq. (3.1) should be checked first at a critical section at $d/2$ from the column face (Fig. 3.1a). If Eq. (3.1) is not satisfied, shear reinforcement is required.

3.3.2 *Shear strength with studs*—The shear strength v_n at a critical section at $d/2$ from the column face should not be taken greater than $6\sqrt{f'_c}$ when stud shear reinforcement is provided. The shear strength at a critical section within the shear-reinforced zone should be computed by:

$$v_n = v_c + v_s \qquad (3.9)$$

in which

$$v_c = 2\sqrt{f'_c} \qquad (3.10)$$

and

$$v_s = \frac{A_v f_{yv}}{b_o s} \qquad (3.11)$$

where A_v is the cross-sectional area of the shear studs on one peripheral line parallel to the perimeter of the column section; s is the spacing between peripheral lines of studs.

The distance s_o between the first peripheral line of shear studs and the column face should not be smaller than $0.35d$. The upper limits of s_o and the spacing s between the peripheral lines should be:

$$s_o \le 0.4d \qquad (3.12)$$

$$s \le 0.5d \qquad (3.13)$$

The upper limit of s_o is intended to eliminate the possibility of shear failure between the column face and the innermost peripheral line of shear studs. Similarly, the upper limit of s is to avoid failure between consecutive peripheral lines of studs.

The shear studs should extend away from the column face so that the shear stress v_u at a critical section at $d/2$ from outermost peripheral line of shear studs [Fig. 3.1(b) and 3.2] does not exceed ϕv_n, where v_n is calculated using Eq. (3.8).

3.4—Design procedure

The values of f'_c, f_{yv}, M_u, V_u, h, and d are given. The design of stud shear reinforcement can be performed by the following steps:

1. At a critical section at $d/2$ from column face, calculate v_u and v_n by Eq. (3.2) and (3.5) to (3.7). If $(v_u/\phi) \le v_n$, no shear reinforcement is required.

2. If $(v_u/\phi) > v_n$, calculate the contribution of concrete v_c to the shear strength [Eq. (3.10)] at the same critical section. The difference $[(v_u/\phi) - v_c]$ gives the shear stress v_s to be resisted by studs.

3. Select s_o and stud spacing s within the limitations of Eq. (3.12) and (3.13), and calculate the required area of stud for one peripheral line A_v, by solution of Eq. (3.11). Find the minimum number of studs per peripheral line.

4. Repeat Step 1 at a trial critical section at αd from column face to find the section where $(v_u/\phi) \le 2\sqrt{f'_c}$. No other

section needs to be checked, and s is to be maintained constant. Select the distance between the column face and the outermost peripheral line of studs to be $\geq (\alpha d - d/2)$.

The position of the critical section can be determined by selection of n_x and n_y (Fig. 3.2), in which n_x and n_y are numbers of studs per line running in x and y directions, respectively. For example, the distance in the x direction between the column face and the critical section is equal to $s_o + (n_x - 1)\, s + d/2$. The two numbers n_x and n_y need not be equal; but each must be ≥ 2.

5. Arrange studs to satisfy the detailing requirements described in Appendix A.

The trial calculations involved in the above steps are suitable for computer use.[14]

CHAPTER 4—PRESTRESSED SLABS

4.1—Nominal shear strength

When a slab is prestressed in two directions, the shear strength of concrete at a critical section at $d/2$ from the column face where stud shear reinforcement is not provided is given by ACI 318:

$$v_n = \beta_p \sqrt{f'_c} + 0.3 f_{pc} + \frac{V_p}{b_o d} \tag{4.1}$$

where β_p is the smaller of 3.5 and $(\alpha_s d/b_o + 1.5)$; f_{pc} is the average value of compressive stress in the two directions (after allowance for all prestress losses) at centroid of cross section; V_p is the vertical component of all effective prestress forces crossing the critical section. Eq. (4.1) is applicable only if the following are satisfied:

a) No portion of the column cross section is closer to a discontinuous edge than four times the slab thickness;

b) f'_c in Eq. (4.1) is not taken greater than 5000 psi; and

c) f_{pc} in each direction is not less than 125 psi, nor taken greater than 500 psi.

If any of the above conditions are not satisfied, the slab should be treated as non-prestressed and Eq. (3.5) to (3.8) apply. Within the shear-reinforced zone, v_n is to be calculated by Eq. (3.9).

In thin slabs, the slope of the tendon profile is hard to control. Special care should be exercised in computing V_p in Eq. (4.1), due to the sensitivity of its value to the as-built tendon profile. When it is uncertain that the actual construction will match design assumption, a reduced or zero value for V_p should be used in Eq. (4.1).

CHAPTER 5—SUGGESTED HIGHER ALLOWABLE VALUES FOR v_c, v_n, s, AND f_{yv}

5.1—Justification

Section 11.5.3 of ACI 318 requires that "stirrups and other bars or wires used as shear reinforcement shall extend to a distance d from extreme compression fiber and shall be anchored at both ends according to Section 12.13 to develop the design yield strength of reinforcement." Test results[1-6] show that studs with anchor heads of area equal to 10 times the cross section area of the stem clearly satisfied that requirement. Further, use of the shear device such as shown in Fig. 1.1 demonstrated a higher shear capacity. Other researchers, as briefly mentioned in Appendix A, successfully applied other configurations. Based on these results, following additions[1] to ACI 318 are proposed to apply when the shear reinforcement is composed of studs with mechanical anchorage capable of developing the yield strength of the rod; the values given in Section 5.2 through 5.5 may be used.

5.2—Value for v_c

The nominal shear strength provided by the concrete in the presence of shear studs, using Eq. (3.9), can be taken as $v_c = 3\sqrt{f'_c}$ instead of $2\sqrt{f'_c}$. Discussion on the design value of v_c is given in Appendix C.

5.3—Upper limit for v_n

The nominal shear strength v_n resisted by concrete and steel in Eq. (3.9) can be taken as high as $8\sqrt{f'_c}$ instead of $6\sqrt{f'_c}$. This enables the use of thinner slabs. Experimental data showing that the higher value of v_n can be used are included in Appendix C.

5.4—Upper limit for s

The upper limits for s can be based on the value of v_u at the critical section at $d/2$ from column face:

$$s \leq 0.75d \text{ when } \frac{v_u}{\phi} \leq 6\sqrt{f'_c} \tag{5.1}$$

$$s \leq 0.5d \text{ when } \frac{v_u}{\phi} > 6\sqrt{f'_c} \tag{5.2}$$

When stirrups are used, ACI 318 limits s to $d/2$. The higher limit for s given by Eq. (5.1) for stud spacing is again justified by tests (see Appendix C).

As mentioned earlier in Chapter 2, a vertical branch of a stirrup is less effective than a stud in controlling shear cracks for two reasons: a) The stud stem is straight over its full length, whereas the ends of the stirrup branch are curved; and b) The anchor plates at the top and bottom of the stud ensure that the specified yield strength is provided at all sections of the stem. In a stirrup, the specified yield strength can be developed only over the middle portion of the vertical legs when they are sufficiently long.

5.5—Upper limit for f_{yv}

Section 11.5.2 of ACI 318 limits the design yield strength for stirrups as shear reinforcement to 60,000 psi. Research[15-17] has indicated that the performance of higher-strength studs as shear reinforcement in slabs is satisfactory. In this experimental work, the stud shear reinforcement in slab-column connections reached yield stress higher than 72,000 psi, without excessive reduction of shear resistance of concrete. Thus, when studs are used, f_{yv} can be as high as 72,000 psi.

CHAPTER 6—TOLERANCES

Shear reinforcement, in the form of stirrups or studs, can be ineffective if the specified distances s_o and s are not controlled accurately. Tolerances for these dimensions should not exceed ± 0.5 in. If this requirement is not met, a punching shear crack can traverse the slab thickness without intersecting the shear reinforcing elements. Tolerance for the distance between column face and outermost peripheral line of studs should not exceed ± 1.5 in.

CHAPTER 7—DESIGN EXAMPLE

The design procedure presented in Chapter 3 is illustrated by a numerical example for an interior column of a non-prestressed slab. A design example for studs at edge column is presented elsewhere.[18] There is divergence of opinions with respect to the treatment of corner and irregular columns.[18-20]

The design of studs is required at an interior column based on the following data: column size c_x by c_y = 12 x 20 in.; slab thickness = 7 in.; concrete cover = 0.75 in.; f'_c = 4000 psi; yield strength of studs f_{yv} = 60 ksi; and flexural reinforcement nominal diameter = 5/8 in. The factored forces transferred from the column to the slab are: V_u = 110 kip and M_{uy} = 50 ft-kip. The five steps of design outlined in Chapter 3 are followed:

Step 1—The effective depth of slab

$$d = 7 - 0.75 - (5/8) = 5.62 \text{ in.}$$

Properties of a critical section at *d/2* from column face shown in Fig. 7.1: b_o = 86.5 in.; A_c = 486 in.^{2}; $J_y = 28.0 \times 10^3$ in.^{4}; l_{x1} = 17.62 in.; l_{y1} = 25.62 in.

The fraction of moment transferred by shear [Eq. (3.3)]:

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{17.62}{25.62}}} = 0.36$$

The maximum shear stress occurs at x = 17.62/2 = 8.81 in. and its value is [Eq. (3.2)]:

$$v_u = \frac{110 \times 1000}{486} + \frac{0.36(50 \times 12{,}000)8.81}{28.0 \times 10^3} = 294 \text{ psi}$$

$$\frac{v_u}{\phi} = \frac{294}{0.85} = 346 \text{ psi} = 5.5\sqrt{f'_c}$$

The nominal shear stress that can be resisted without shear reinforcement at the critical section considered [Eq. (3.5) to (3.7)]:

$$v_n = \left(2 + \frac{4}{1.67}\right)\sqrt{f'_c} = 4.4\sqrt{f'_c}$$

$$v_n = \left[\frac{40(5.62)}{86.5} + 2\right]\sqrt{f'_c} = 4.6\sqrt{f'_c}$$

$$v_n = 4\sqrt{f'_c}$$

use the smallest value: $v_n = 4\sqrt{f'_c}$ = 253 psi

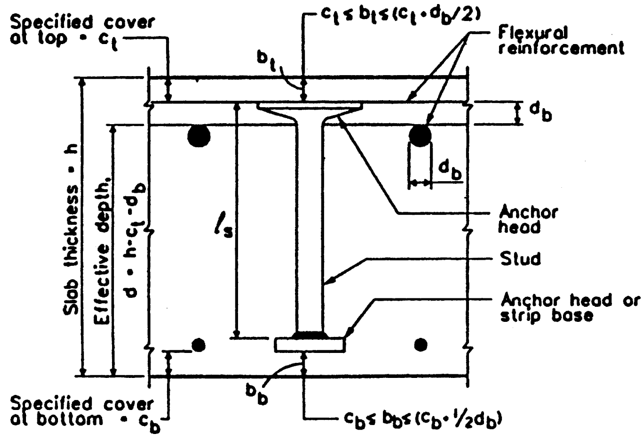


Fig. 7.1—Section in slab perpendicular to shear stud line.

Step 2—The quantity v_n/ϕ is greater than v_n, indicating that shear reinforcement is required; the same quantity is less than the upper limit $v_n = 6\sqrt{f'_c}$, which means that the slab thickness is adequate.

The shear stress resisted by concrete in the presence of the shear reinforcement (Eq. 3.10) at the same critical section:

$$v_c = 2\sqrt{f'_c} = 126 \text{ psi}$$

Use of Eq. (3.1), (3.9), and (3.11) gives:

$$v_s \ge \frac{v_u}{\phi} - v_c = 346 - 126 = 220 \text{ psi}$$

$$\frac{A_v}{s} \ge \frac{v_s b_o}{f_{yv}} = \frac{220(86.5)}{60{,}000} = 0.32 \text{ in.}$$

Step 3—

$$s_o \le 0.4\,d = 2.25 \text{ in.}; s \le 0.5d = 2.8 \text{ in.}$$

This example has been provided for one specific type of shear stud reinforcement, but the approach can be adapted and used also for other types mentioned in Appendix A.

Try 3/8 in. diameter studs welded to a bottom anchor strip 3/16 x 1 in. Taking cover of 3/4 in. at top and bottom, the length of stud l_s (Fig. 7.1) should not exceed:

$$l_{smax} = 7 - 2\left(\frac{3}{4}\right) - \frac{3}{16} = 5\frac{5}{16} \text{ in.}$$

or the overall height of the stud, including the two anchors, should not exceed 5.5 in.

Also, l_s should not be smaller than:

$$l_{smin} = l_{smax} - \text{one bar diameter of flexural reinforcement}$$

$$l_{smin} = 5\frac{5}{16} - \frac{5}{8} = 4\frac{11}{16} \text{ in.}$$

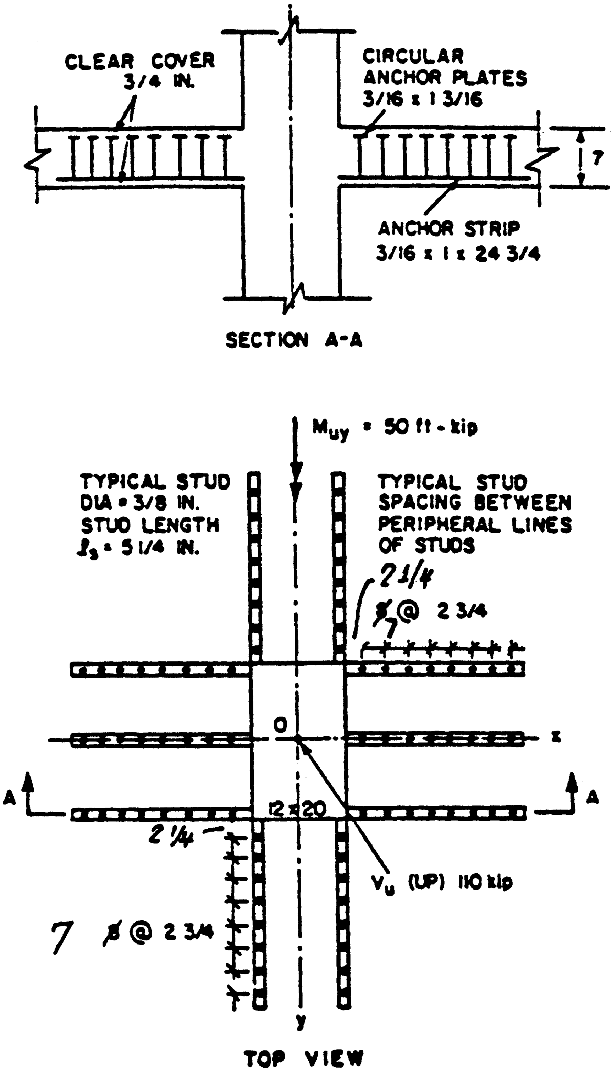


Fig. 7.2—Example of stud arrangement.

Choose l_s = 5-1/4 in. With 10 studs per peripheral line, choose the spacing between peripheral lines, s = 2.75 in., and the spacing between column face and first peripheral line, s_o = 2.25 in. (Fig. 7.2).

$$\frac{A_v}{s} = \frac{10(0.11)}{2.75} = 0.4 \text{ in.}$$

This value is greater than 0.32 in., indicating that the choice of studs and their spacing is adequate.

Step 4—Properties of critical section at 4d from column face [Fig. 3.1(b)]:

$\alpha = 4.0$; $\alpha d = 4(5.62) = 22.5$ in.;

$l_{x1} = 14.3$ in.; $l_{y1} = 22.3$ in.;

$l_{x2} = 57.0$ in.; $l_{y2} = 65.0$ in.;

$l = 30.2$ in.; $b_o = 194.0$ in.;

$A_c = 1090$ in.^{2}; $J_y = 449.5 \times 10^3$ in.^{4}.

The maximum shear stress in the critical section occurs on line AB at:

$x = 57/2 = 28.5$ in.; Eq. (3.2) gives:

$$v_u = \frac{110{,}000}{1090} + \frac{0.36(50 \times 12{,}000)28.5}{449.5 \times 10^3} = 115 \text{ psi}$$

$$\frac{v_u}{\phi} = \frac{115}{0.85} = 135 \text{ psi}$$

$$v_n = 2\sqrt{f'_c} = 126 \text{ psi}$$

The value $v_u/\phi = 135$ psi is greater than $v_n = 126$ psi, which indicates that shear stress should be checked at $\alpha > 4$. Try eight peripheral lines of studs; distance between column face and outermost peripheral line of studs:

$$\alpha d = s_o + 7s = 2.25 + 7(2.75) = 22 \text{ in.}$$

Check shear stress at a critical section at a distance from column face:

$$\alpha d = 22 + d/2 = 22.0 + 5.62/2 = 24.8 \text{ in.}$$

$$\alpha = \frac{24.8}{d} = \frac{24.8}{5.62} = 4.4$$

$$v_u = 125 \text{ psi}$$

$$v_n = 2\sqrt{f'_c} = 126 \text{ psi}$$

Step 5—The value of v_u/ϕ is less than v_n, which indicates that details of stud arrangement as shown in Fig. 7.2 are adequate.

The value of V_u used to calculate the maximum shear stress could have been reduced by the counteracting factored load on the slab area enclosed by the critical section.

If the higher allowable values of v_c and s proposed in Chapter 5 are adopted in this example, it will be possible to use only six peripheral lines of studs instead of eight, with spacing s = 4.0 in., instead of 2.75 in. used in Fig. 7.2.

CHAPTER 8—REQUIREMENTS FOR SEISMIC-RESISTANT SLAB-COLUMN CONNECTIONS IN REGIONS OF SEISMIC RISK

Connections of columns with flat plates should not be considered in design as part of the system resisting lateral forces. However, due to the lateral movement of the structure in an earthquake, the slab-column connections transfer vertical shearing force V combined with reversal of moment M. Experiments[21-23] were conducted on slab-column connections to simulate the effect of interstorey drift in a flat-slab structure. In these tests, the column was transferring a constant shearing force V and cyclic moment reversal with increasing magnitude. The experiments showed that, when the slab is provided with stud shear reinforcement the connections behave in a ductile fashion. They can withstand, without failure, drift ratios varying between 3 and 7%, depending upon the magnitude of V. The drift ratio is defined as the difference between the lateral displacements of two successive floors divided by the floor height. For a given value V_u, the slab can resist a moment M_u, which can be determined by the procedure and equations given in Chapters 3 and 5; but the value of v_c should be limited to:

$$v_c = 1.5\sqrt{f'_c} \tag{8.1}$$

This reduced value of v_c is based on the same experiments, which indicate that the concrete contribution to the shear resistance is diminished by the moment reversals. This reduction is analogous to the reduction of v_c to 0 by Section 21.3.4.2 of ACI 318 for framed members.

CHAPTER 9—REFERENCES

9.1—Recommended references

The documents of the various standards-producing organizations referred to in this document are listed below with their serial designation.

American Concrete Institute

318/318R Building Code Requirements for Structural Concrete and Commentary

British Standards Institution

BS 8110 Structural Use of Concrete

Canadian Standards Association

CSA-A23.3 Design of Concrete Structures for Buildings

The above publications may be obtained from the following organizations:

American Concrete Institute
P.O. Box 9094
Farmington Hills, MI 48333-9094

British Standards Institution
2 Park Street
London W1A 2BS
England

Canadian Standards Association
178 Rexdale Blvd.
Rexdale, Ontario M9W 1R3
Canada

9.2—Cited references

1. Dilger, W. H., and Ghali, A., “Shear Reinforcement for Concrete Slabs,” *Proceedings*, ASCE, V.107, ST12, Dec. 1981, pp. 2403-2420.

2. Andrä, H. P., “Strength of Flat Slabs Reinforced with Stud Rails in the Vicinity of the Supports (Zum Tragverhalten von Flachdecken mit Dübelleisten-Bewehrung im Auflagerbereich),” *Beton und Stahlbetonbau*, Berlin, V. 76, No. 3, Mar. 1981, pp. 53-57, and No. 4, Apr. 1981, pp. 100-104.

3. Mokhtar, A. S.; Ghali, A.; and Dilger, W. H., “Stud Shear Reinforcement for Flat Concrete Plates,” ACI JOURNAL, *Proceedings* V. 82, No. 5, Sept.-Oct. 1985, pp. 676-683.

4. Elgabry, A. A., and Ghali, A., “Tests on Concrete Slab-Column Connections with Stud Shear Reinforcement Subjected to Shear-Moment Transfer,” *ACI Structural Journal,* V. 84, No. 5, Sept.-Oct. 1987, pp. 433-442.

5. Mortin, J., and Ghali, A., “Connection of Flat Plates to Edge Columns,” *ACI Structural Journal*, V. 88, No. 2, Mar.-Apr. 1991, pp. 191-198.

6. Dilger, W. H., and Shatila, M., “Shear Strength of Prestressed Concrete Edge Slab-Columns Connections with and without Stud Shear Reinforcement,” *Concrete Journal of Civil Engineering*, V. 16, No. 6, 1989, pp. 807-819.

7. U.S. patent No. 4406103. Licensee: Deha, represented by Decon, Medford, NJ, and Brampton, Ontario.

8. Zulassungsbescheid Nr. Z-4.6-70, “Kopfbolzen-Dübbelleisten als Schubbewehrung im Stützenbereich punkfürmig gestützter Platten (Authorization No. Z-4.6-70, (Stud Rails as Shear Reinforcement in the Support Zones of Slabs with Point Supports),” Berlin, Institut fur Bautechnik, July 1980.

9. Regan, P. E., “Shear Combs, Reinforcement against Punching,” *The Structural Engineer*, V. 63B, No. 4, Dec. 1985, London, pp. 76-84.

10. Marti, P., “Design of Concrete Slabs for Transverse Shear,” *ACI Structural Journal*, V. 87, No. 2, Mar.-Apr. 1990, pp. 180-190.

11. ASCE-ACI Committee 426, “The Shear Strength of Reinforced Concrete Members-Slabs,” *Journal of the Structural Division*, ASCE, V. 100, No. ST8, Aug. 1974, pp. 1543-1591.

12. Hawkins, N. M., “Shear Strength of Slabs with Shear Reinforcement,” *Shear in Reinforced Concrete,* SP-42, American Concrete Institute, Farmington Hills, Mich., 1974, pp. 785-815.

13. Hawkins, N. M.; Mitchell, D.; and Hanna, S. H., “The Effects of Shear Reinforcement on Reversed Cyclic Loading Behavior of Flat Plate Structures,” *Canadian Journal of Civil Engineering,* V. 2, No. 4, Dec. 1975, pp. 572-582.

14. Decon, “STDESIGN,” Computer Program for Design of Shear Reinforcement for Slabs, 1996, Decon, Brampton, Ontario.

15. Otto-Graf-Institut, “Durchstanzversuche an Stahlbetonplatten (Punching Shear Research on Concrete Slabs),” *Report* No. 21-21634, Stuttgart, Germany, July 1996.

16. Regan, P. E., “Double Headed Studs as Shear Reinforcement—Tests of Slabs and Anchorages,” University of Westminster, London, Aug. 1996.

17. “Bericht über Versuche an punktgestützten Platten bewehrt mit DEHA Doppelkopfbolzen und mit Dübelleisten (Test Report on Point Supported Slabs Reinforced with DEHA Double Head Studs and Studrails),” Institut für Werkstoffe im Bauwesen, *Universität—Stuttgart, Report* No. AF 96/6 - 402/1, DEHA 1996, 81 pp.

18. Elgabry, A. A., and Ghali, A., “Design of Stud Shear Reinforcement for Slabs,” *ACI Structural Journal*, V. 87, No. 3, May-June 1990, pp. 350-361.

19. Rice, P. F.; Hoffman, E. S.; Gustafson, D. P.; and Gouwens, A. I., *Structural Design Guide to the ACI Building Code*, 3rd Edition, Van Nostrand Reinhold, New York.

20. Park, R., and Gamble, W. L., *Reinforced Concrete Slabs,* J. Wiley & Sons, New York, 1980, 618 pp.

21. Brown, S. and Dilger, W. H., “Seismic Response of Flat-Plate Column Connections,” *Proceedings*, Canadian Society for Civil Engineering Conference, V. 2, Winnipeg, Manitoba, Canada, 1994, pp. 388-397.

22. Cao, H., “Seismic Design of Slab-Column Connections,” MSc thesis, University of Calgary, 1993, 188 pp.

23. Megally, S. H., “Punching Shear Resistance of Concrete Slabs to Gravity and Earthquake Forces,” PhD dissertation, University of Calgary, 1998, 468 pp.

24. Dyken T., and Kepp, B., “Properties of T-Headed Reinforcing Bars in High-Strength Concrete,” *Publication* No. 7, Nordic Concrete Research, Norske Betongforening, Oslo, Norway, Dec. 1988.

25. Hoff, G. C., “High-Strength Lightweight Aggregate Concrete—Current Status and Future Needs,” *Proceedings,* 2nd International Symposium on Utilization of High-Strength Concrete, Berkeley, Calif., May 1990, pp. 20-23.

26. McLean, D.; Phan, L. T.; Lew, H. S.; and White, R. N., “Punching Shear Behavior of Lightweight Concrete Slabs and Shells,” *ACI Structural Journal,* V. 87, No. 4, July-Aug. 1990, pp. 386-392.

27. Muller, F. X.; Muttoni, A.; and Thurlimann, B., “Durchstanz Versuche an Flachdecken mit Aussparungen (Punching Tests on Slabs with Openings),” ETH Zurich, *Research Report* No. 7305-5, Birkhauser Verlag, Basel and Stuttgart, 1984.

28. Mart, P.; Parlong, J.; and Thurlimann, B., *Schubversuche and Stahlbeton-Platten*, Institut fur Baustatik aund Konstruktion, ETH Zurich, *Bericht* Nr. 7305-2, Birkhauser Verlag, Basel und Stuttgart, 1977.

29. Ghali, A.; Sargious, M. A.; and Huizer, A., “Vertical Prestressing of Flat Plates around Columns,” *Shear in Reinforced Concrete,* SP-42, Farmington Hills, Mich., 1974, pp. 905-920.

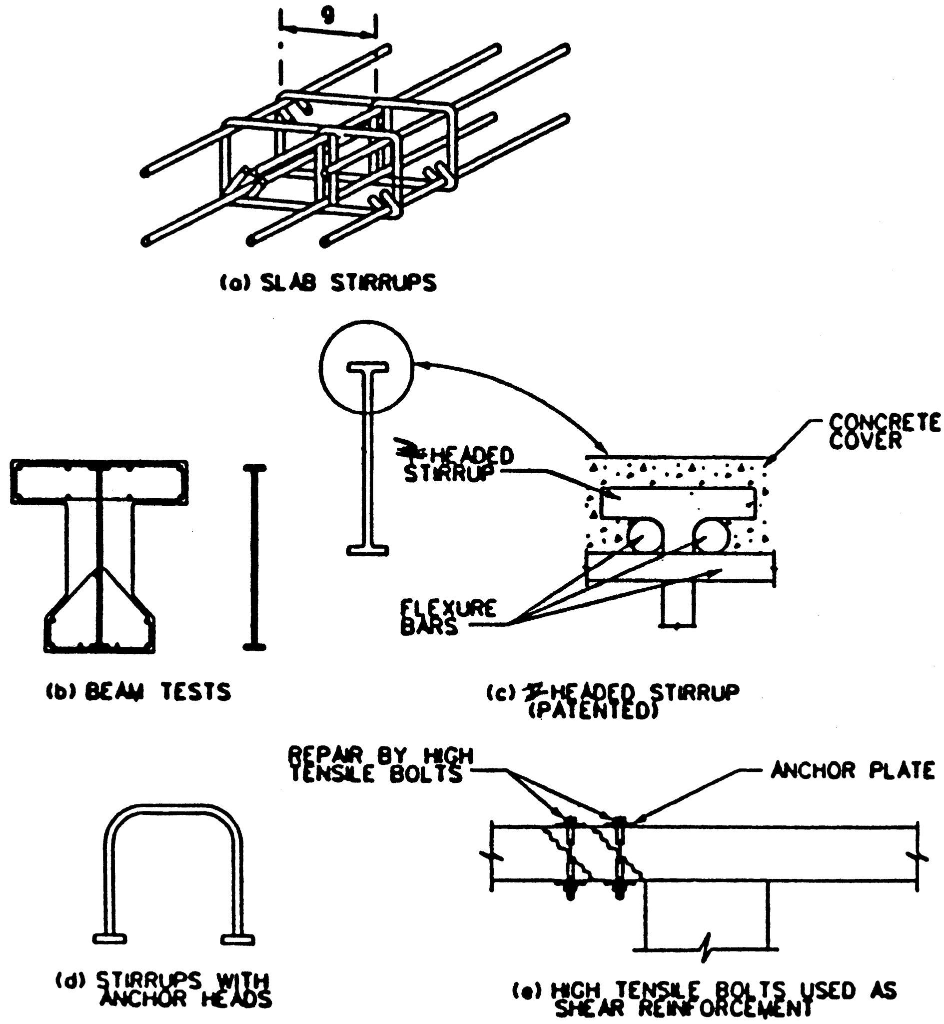


Fig. A1—Shear reinforcement types (a) to (e) are from ACI 318 and cited References 24, 26, 27, and 29, respectively.

30. Elgabry, A. A., and Ghali, A., "Moment Transfer by Shear in Slab-Column Connections," *ACI Structural Journal*, V. 93, No. 2, Mar.-Apr. 1996, pp. 187-196.

31. Megally, S., and Ghali, A., "Nonlinear Analysis of Moment Transfer between Columns and Slabs," *Proceedings*, V. IIa, Canadian Society for Civil Engineering Conference, Edmonton, Alberta, Canada, 1996, pp. 321-332.

32. Leonhardt, F., and Walter, R., "Welded Wire Mesh as Stirrup Reinforcement: Shear on T-Beams and Anchorage Tests," Bautechnik, V. 42, Oct. 1965. (in German)

33. Andrä, H.-P., "Zum Verhalten von Flachdecken mit Dübelleisten—Bewehrung im Auglagerbereich (On the Behavior of Flat Slabs with Stud-rail Reinforcement in the Support Region)," *Beton und Stahlbetonbau* 76, No. 3, pp. 53-57, and No. 4, pp. 100-104, 1981.

34. "Durchstanzversuche an Stahlbetonplatten mit Rippendübeln und Vorgefertigten Gross-flächentafeln (Punching Shear Tests on Concrete Slabs with Deformed Studs and Large Precast Slabs)," *Report* No. 21-21634, Otto-Graf-Institut, University of Stuttgart, July 1996.

35. Regan, P. E., "Punching Test of Slabs with Shear Reinforcement," University of Westminster, London, Nov. 1996.

36. Sherif, A., "Behavior of R.C. Flat Slabs," PhD dissertation, University of Calgary, 1996, 397 pp.

37. Van der Voet, F.; Dilger, W.; and Ghali, A., "Concrete Flat Plates with Well-Anchored Shear Reinforcement Elements," *Canadian Journal of Civil Engineering*, V. 9, 1982, pp. 107-114.

38. Elgabry, A., and Ghali, A., "Tests on Concrete Slab-Column Connections with Stud-Shear Reinforcement Subjected to Shear Moment Transfer," *ACI Structural Journal*, V. 84, No. 5, Sept.-Oct. 1987, pp. 433-442.

39. Seible, F.; Ghali, A.; and Dilger, W., "Preassembled Shear Reinforcing Units for Flat Plates," ACI JOURNAL, *Proceedings* V. 77, No. 1, Jan.-Feb. 1980, pp. 28-35.

APPENDIX A—DETAILS OF SHEAR STUDS

A.1—Geometry of stud shear reinforcement

Several types and configurations of shear studs have been reported in the literature. Shear studs mounted on a continuous steel strip, as discussed in the main text of this report, have been developed and investigated.[1-6] Headed reinforcing bars were developed and applied in Norway[24] for high-strength concrete structures, and it was reported that such applications improved the structural performance significantly.[25] Another type of headed shear reinforcement was implemented for increasing the punching shear strength of lightweight concrete slabs and shells.[26] Several other approaches for mechanical anchorage in shear reinforcement can be used.[10, 27-29] Several types are depicted in Fig. A1; the figure also shows the required details of stirrups when used in slabs according to ACI 318R.

The anchors should be in the form of circular or rectangular plates, and their area must be sufficient to develop the specified yield strength of studs f_{yv}. It is recommended that

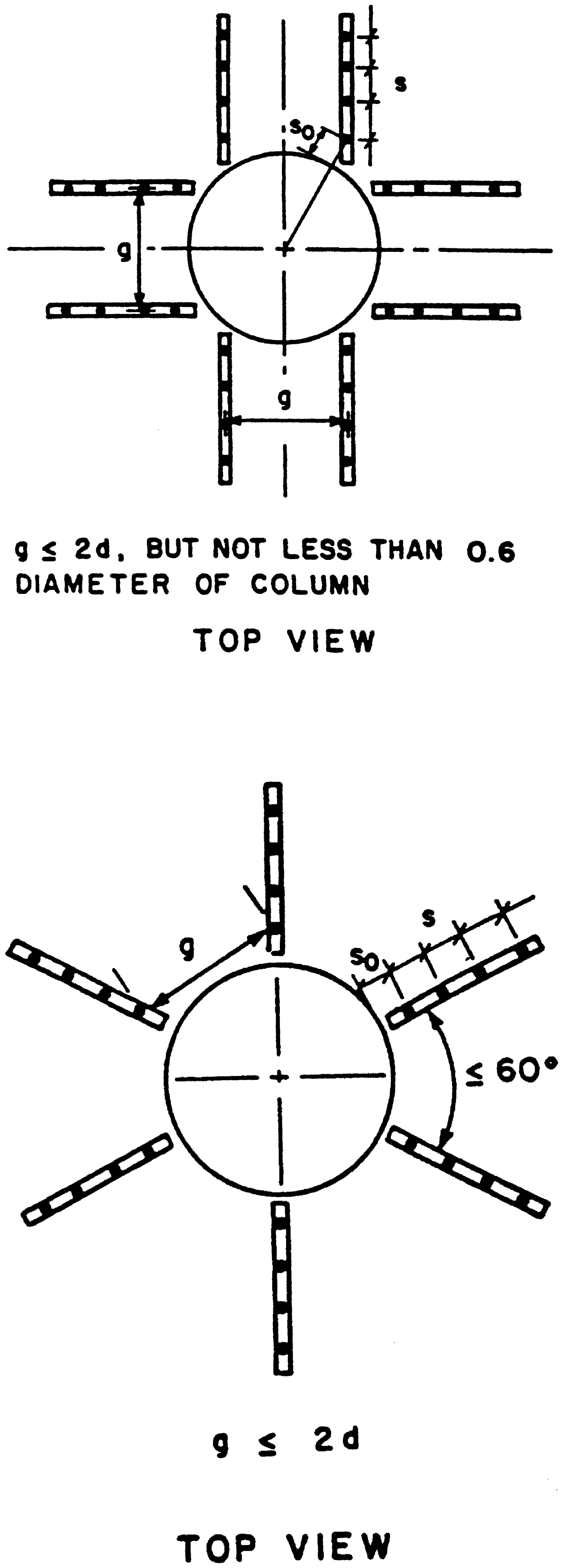


Fig. A2— Stud shear reinforcement arrangement for circular columns.

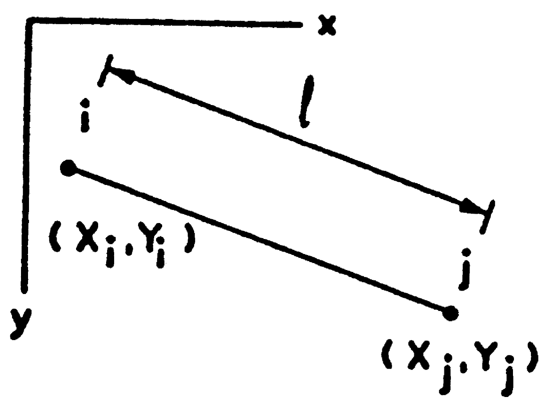


Fig. B1—Straight line representing typical segment of critical section perimeter. Definition of symbols used in Eq. (B-1) to (B-3).

the performance of the shear stud reinforcement be verified before their use. The user can find such information in the cited references.

A.2—Stud arrangements

Shear studs in the vicinity of rectangular columns should be arranged on peripheral lines. The term peripheral line is used in this report to mean a line running parallel to and at constant distance from the sides of the column cross section. Fig. 3.2 shows a typical arrangement of stud shear reinforcement in the vicinity of a rectangular interior, edge, and corner columns. Tests[1] showed that studs are most effective near column corners. For this reason, shear studs in Fig. 3.2(a), (b), and (c) are aligned with column faces. In the direction parallel to a column face, the distance g between lines of shear studs should not exceed $2d$, where d is the effective depth of the slab. When stirrups are used, the same limit for g should be observed [Fig. A1(a)].

The stud arrangements for circular columns are shown in Fig. A2. The minimum number of peripheral lines of shear studs, in the vicinity of rectangular and circular columns, is two.

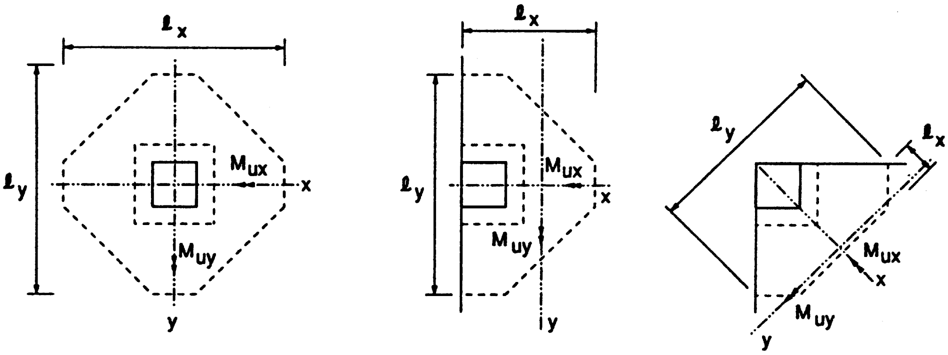
A.3—Stud length

The studs are most effective when their anchors are as close as possible to the top and bottom surfaces of the slab. Unless otherwise protected, the minimum concrete cover of the anchors should be as required by Section 7.7 of ACI 318. The cover of the anchors should not exceed the minimum cover plus one half bar diameter of flexural reinforcement (Fig. 7.1). The mechanical anchors should be placed in the forms above reinforcement supports, which insure the specified concrete cover.

APPENDIX B—PROPERTIES OF CRITICAL SECTIONS OF GENERAL SHAPE

This appendix is general; it applies regardless of the type of shear reinforcement used. Fig. 3.1 shows the top view of critical sections for shear in slab in the vicinity of interior column. The centroidal x and y axes of the critical sections, V_u, M_{ux}, and M_{uy} are shown in their positive directions. The shear force V_u is acting at the column centroid; V_u, M_{ux}, and M_{uy} represent the effects of the column on the slab.

In use of Eq. (3.2), v_u for a section of general shape (for example, Fig. 3.1 and 3.2), the parameters J_x and J_y may



$$\gamma_{vx} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\ell_y/\ell_x}}$$

γ_{vx} = same as for interior column

$\gamma_{vx} = 0.4$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\ell_x/\ell_y}}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{(\ell_x/\ell_y) - 0.2}}$$

(but $\gamma_{vy} = 0$ when $(\ell_x/\ell_y) < 0.2$)

γ_{vy} = same as for edge column

Fig. B2—Equations for γ_{vx} and γ_{vy} applicable for critical sections at d/2 from column face and outside shear-reinforced zone. Note: l_x and l_y are projections of critical sections on directions of principal x and y axes.

Table C1—List of references on slab-column connections tests using stud shear reinforcement

Experiment no.	Reference no.	Experiment no.	Reference no.	Experiment no.	Reference no.
1 to 5	33	16 to 18	16	26 to 29	38
6, 7	See footnote*	19 to 20	35	30 to 36	3
8, 9	34	21 to 24, 37	36	42	39
10 to 15	17	25, 38 to 41	37	—	—

* "Grenzzustände der Tragfäkigheit für Durchstanzen von Platten mit Dübelleistein nach EC2 (Ultimate Limit States of Punching of Slabs with Studrails According to EC2)," Private communication with Leonhardt, Andrä, and Partners, Stuttgart, Germany, 1996, 15 pp.

Table C2—Slabs with stud shear reinforcement failing within shear-reinforced zone

Experiment	Square column size, in.	f_c', in.	d, in.	s/d	Tested capacities V_u, kip	Tested capacities M_u, kip-in.	M at critical section centroid, kip-in.	Maximum shear stress v_u, psi	f_{yv}	A_v, in.^{2}	V_{test}/V_{code}†	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
20	7.87	5660	6.30	0.75	214	0	0	599	64.1	1.402	1.00	Interior column
21	9.84	4100	4.49	0.70	47.4	651	491	528	55.1	0.66	1.14	Edge column
22	9.84	4030	4.49	0.70	52.8	730	552	590	55.1	0.66	1.28	Edge column
23	9.84	4080	4.49	0.70	26.0	798	708	641	55.1	0.66	1.39	Edge column
24	9.84	4470	4.49	0.70	27.2	847	755	693	55.1	0.66	1.48	Edge column
26	9.84	4890	4.49	0.75	34	1434	1434	570	66.7	1.570	1.02	Interior column
27	9.84	5660	4.49	0.75	67	1257	1257	641	66.7	1.570	1.06	Interior column
28	9.84	5920	4.49	0.5 and 0.95	67	1328	1328	665	66.7	0.880	1.08	Interior column
29	9.84	6610	4.49	0.5 and 0.97	101	929	929	673	66.7	0.880	1.03	Interior column
30*	9.84	5470	4.49	0.75	117	0	0	454	40.3	1.320	1.02	Interior column
39	9.84	4210	4.45	0.88	113	0	0	444	47.1	0.460	1.52	Interior column
								Mean			1.18	
								Coefficient of variation			0.17	

*Semi-lightweight concrete; $\sqrt{f_c'}$ is replaced in calculation by $f_{ct}/6.7$; f_{ct} is average splitting tensile strength of lightweight aggregate concrete; f_{ct} used here = 377 psi, determined experimentally.

†v_{code} is smaller of $8\sqrt{f_c'}$ and $(3\sqrt{f_c'} + v_s)$; where $v_s = A_v f_{yv}/(b_o s)$.

Table C3—Experiments with maximum shear stress v_u at critical section of $d/2$ from column face exceeding $8\sqrt{f_c'}$ (slabs with stud shear reinforcement)

Experiment	Column size, in.*	f_c', psi	$8\sqrt{f_c'}$, psi	Tested capacities		d, in.	M at critical section centroid, kip-in.	Maximum shear stress v_u, psi	$v_u/8\sqrt{f_c'}$
				V, kip	M, kip-in.				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	11.81 sq.	6020	621	476	0	9.06	0	629	1.07
2	11.81 sq.	5550	589	428	0	8.86	0	585	1.00
3	11.81 sq.	3250	456	346	0	8.66	0	488	1.07
4	19.68 cr.	5550	589	665	0	10.51	0	667	1.13
5	14.57 sq.	6620	651	790	0	11.22	0	682	1.05
6	12.60 cr.	5870	613	600	0	9.33	0	934	1.52
7	12.60 cr.	6020	621	620	0	9.33	0	965	1.55
8	10.23 sq.	3120	447	271	0	8.07	0	459	1.03
9	10.23 sq.	3270	457	343	0	8.07	0	582	1.27
10	7.48 cr.	3310	460	142	0	5.83	0	582	1.26
11	7.48 cr.	3260	456	350	0	9.60	0	679	1.48
12	7.48 cr.	4610	543	159	0	6.02	0	623	1.14
13	7.48 cr.	3050	441	128	0	5.91	0	516	1.17
14	7.48 cr.	3340	462	278	0	9.72	0	530	1.14
15	7.48 cr.	3160	449	255	0	9.76	0	482	1.07
16	9.25 cr.	4630	544	207	0	5.94	0	728	1.34
17	9.25 cr.	5250	580	216	0	6.14	0	725	1.25
18	9.25 cr.	5290	582	234	0	6.50	0	725	1.24
19	7.87 sq.	5060	569	236	0	6.30	0	661	1.16
20	7.87 sq.	5660	601	214	0	6.30	0	599	1.00
21†	9.84 sq.	4100	513	47.4	651	4.49	491	528	1.03
22†	9.84 sq.	4030	508	52.8	730	4.49	552	590	1.16
23†	9.84 sq.	4080	511	26.9	798	4.49	708	641	1.25
24†	9.84 sq.	4470	535	27.2	847	4.49	755	693	1.29
25	9.84 sq.	4280	523	135	0	4.45	0	532	1.02
26	9.84 sq.	4890	559	33.7	1434	4.49	1434	570	1.02
27	9.84 sq.	5660	602	67.4	1257	4.49	1257	641	1.06
28	9.84 sq.	5920	615	67.4	1328	4.49	1328	665	1.08
29	9.84 sq.	6610	651	101	929	4.49	924	673	1.03
							Mean		1.17
							Coefficient of variation		0.13

*Column 2 gives side dimension of square (sq.) columns or diameter of circular (cr.) columns.
†Edge slab-column connections. Other experiments are on interior slab-column connections.

be approximated by the second moments of area I_x and I_y given in Eq. (B-2) and (B-3). The coefficients γ_{vx} and γ_{vy} are given in Fig. B2, which is based on finite element studies.[30,31]

The critical section perimeter is generally composed of straight segments. The values of A_c, I_x, and I_y can be determined by summation of the contribution of the segments (Fig. 3.2):

$$A_c = d\sum l \tag{B-1}$$

$$I_x = d\sum\left[\frac{l}{3}(y_i^2 + y_i y_j + y_j^2)\right] \tag{B-2}$$

$$I_y = d\sum\left[\frac{l}{3}(x_i^2 + x_i x_j + x_j^2)\right] \tag{B-3}$$

where x_i, y_i, x_j, and y_j are coordinates of Points I and j at the extremities of the segment whose length is l (Fig. B1).

When the maximum v_u occurs at a single point on the critical section, rather than on a side, the peak value of v_u does not govern the strength due to stress redistribution.[21] In this case, v_u may be investigated at a point located at a distance $0.4d$ from the peak point. This will give a reduced v_u value compared with the peak value; the reduction should not be allowed to exceed 15%.

APPENDIX C—VALUES OF v_c WITHIN SHEAR REINFORCED ZONE

This design procedure of the shear reinforcement requires calculation of $v_n = v_c + v_s$ at the critical section at $d/2$ from the column face. The value allowed for v_c is $2\sqrt{f_c'}$ when stirrups are used, and $3\sqrt{f_c'}$ when shear studs are used. The reason for the higher value of v_c for slabs with shear stud

Table C4—Slabs with stud shear reinforcement having *s* approximately equal to or greater than 0.75*d*

Experiment	Column size,† in.	f'_c,‡ psi	d, in.	s/d	Tested capacities V, kip	Tested capacities M, kip-in.	M at critical section centroid, kip-in.	Maximum shear stress v_u,§ psi	f_{yv}	A_v, in²	$(v_u)_{outside}$,‖ psi	V_{test}/V_{code}**
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
3	11.81 sq.	3250	8.66	0.55 and 0.73	346	0	0	488	47.9	?	214	1.77
12	7.48 cr.	4610	6.02	0.75	159	0	0	623	67.6	1.09	195	1.42
13	7.48 cr.	3050	5.91	0.77	128	0	0	517	67.6	1.09	160	1.43
16	9.25 cr.	4630	5.94	0.66	207	0	0	728	72.5	1.46	182	1.34
17	9.25 cr.	5250	6.14	0.65	216	0	0	725	72.5	1.46	180	1.26
18	9.25 cr.	5290	6.50	0.61	234	0	0	725	42.5	1.46	181	1.26
19	7.87 sq.	5060	6.30	0.75	236	0	0	661	54.1	1.40	165	1.08
21	9.84 sq.	4100	4.49	0.70	47.4	651	491	528	55.1	0.66	—	1.07
22	9.84 sq.	4030	4.49	0.70	52.8	730	552	590	55.1	0.66	—	1.20
23	9.84 sq.	4080	4.49	0.70	26.9	798	708	641	55.1	0.66	—	1.30
24	9.84 sq.	4470	4.49	0.70	27.2	847	755	693	55.1	0.66	—	1.38
26	9.84 sq.	4890	4.49	0.75	33.7	1434	1434	570	66.7	1.57	—	1.02
27	9.84 sq.	5660	4.49	0.75	67.4	1257	1257	641	66.7	1.57	—	1.06
30*	9.84 sq.	5470	4.49	0.75	117	0	0	454	40.3	1.32	—	1.02
31	9.84 sq.	3340	4.49	0.75	123	0	0	476	40.3	1.32	136	1.18
32	9.84 sq.	5950	4.49	0.75	131	0	0	509	70.9	1.32	145	0.94
33	9.84 sq.	5800	4.49	0.75	131	0	0	509	40.3	1.32	145	0.95
34	9.84 sq.	4210	4.49	0.75	122	0	0	473	70.9	1.32	166	1.28
35	9.84 sq.	5080	4.49	0.75 and 1.50	129	0	0	500	40.3	1.32	143	1.00
36	9.84 sq.	4350	4.49	0.75	114	0	0	444	70.9	1.32	178	1.35
38	9.84 sq.	4790	4.49	0.70	48	637	476	522	55.1	0.66	—	1.03
39	9.84 sq.	4210	4.45	0.88	113	0	0	444	47.1	0.46	—	1.52
40	9.84 sq.	4240	4.45	1.00	125	0	0	492	52.3	1.74	253	1.94
41	9.84 sq.	5300	4.45	0.88	133	0	0	523	49.2	0.99	221	1.52
42	9.84 sq.	5380	4.45	0.88	133	0	0	523	49.2	1.48	273	1.86
43	12.0 sq.	4880	4.76	1.00	134	0	0	419	73.0	1.54	270	1.93
								Mean				1.31
								Coefficient of variation				0.23

*Slab 30 is semi-lightweight. Concrete $\sqrt{f'_c}$ replaced in calculations by $f_{ct}/6.7$; f_{ct} average splitting tensile strength of lightweight aggregate concrete; f_{ct} used here = 377 psi, determined experimentally.

†Column 2 gives side dimension of square (sq.) columns, or diameter of circular (cr.) columns.

‡For cube strengths, concrete cylinder strength in Column 3 calculated using $f'_c = 0.83 f'_{cube}$.

§Column 9 is maximum shear stress at failure in critical section at $d/2$ from column face.

‖$(v_u)_{outside}$ in Column 12 is maximum shear stress at failure in critical section at $d/2$ outside outermost studs; $(v_u)_{outside}$ not given for slabs that failed within stud zone.

**v_{code} is value allowed by ACI 318 combined with proposed equations in Chapter 5. v_{code} calculated at $d/2$ from column face when failure is within stud zone and at section at $d/2$ from outermost studs when failure is outside shear-reinforced zone.

reinforcement is the almost slip-free anchorage of the studs. In structural elements reinforced with conventional stirrups, the anchorage by hooks or 90-deg bends is subject to slip, which can be as high as 0.04 in. when the stress in the stirrup leg approaches its yield strength.[32] This slip is detrimental to the effectiveness of stirrups in slabs because of their relative small depth compared with beams. The influence of the slip is manifold:

- Increase in width of the shear crack;
- Extension of the shear crack into the compression zone;
- Reduction of the shear resistance of the compression zone; and
- Reduction of the shear friction across the crack.

All of these effects reduce the shear capacity of the concrete in slabs with stirrups. To reflect the stirrup slip in the shear resistance equations, refinement of the shear failure model is required. The empirical equation $v_n = v_c + v_s$, adopted in almost all codes, is not the ideal approach to solve the shear design problem. A mechanics-based model that is acceptable for codes is not presently available. There is, however, enough experimental evidence that use of the empirical equation $v_n = v_c + v_s$ with $v_c = 3\sqrt{f'_c}$ gives a safe design for slabs with stud shear reinforcement. This approach is adopted in Canadian code (CSA 23.3).

Numerous test slab-column connections reinforced with shear studs are reported in the literature (Table C1). In the majority of these, the failure is at sections outside the shear-reinforced zone. Table C2 lists only the tests in which the failure occurred within the shear-reinforced zone. Column 12 of Table C2 gives the ratio v_{test}/v_{code}; where v_{code} is the value allowed by ACI 318, but with $v_c = 3\sqrt{f_c'}$ (instead of $2\sqrt{f_c'}$). The values of v_{test}/v_{code} being greater than 1.0 indicate there is safety of design with $v_c = 3\sqrt{f_c'}$.

Table C3 summarizes experimental data of numerous slabs in which the maximum shear stress v_u obtained in test, at the critical section at $d/2$ from column face, reaches or exceeds $8\sqrt{f_c'}$. Table C3 indicates that v_n can be safely taken equal to $8\sqrt{f_c'}$ (Section 5.3).

Table C4 gives the experimental results of slabs having stud shear reinforcement with the spacing between studs greater or close to the upper limit given by Eq. (5.1). In Table C4, v_{code} is the nominal shear stress calculated by ACI 318, combined with the provisions suggested in Chapter 5. The value v_{code} is calculated at $d/2$ from column face when failure is within the shear-reinforced zone, or at a section at $d/2$ from the outermost studs when failure occurs outside the shear-reinforced zone. The ratio v_{test}/v_{code} being greater than 1.0 indicates that it is safe to use studs spaced at the upper limit set by Eq. (5.1) and calculate the strength according to ACI 318 combined with the provisions in Chapter 5.